Seventh International Conference on Construction in the 21st Century (CITC-VII)
"Challenges in Innovation, Integration and Collaboration in Construction & Engineering"
December 19-21, 2013, Bangkok, Thailand

Probabilistic method to predict activity duration for critical path method with increased reliability

Nirdosha Gamage, Guomin Zhang, Chun-Qing Li and Sujeeva Setunge
School of Civil, Environmental and Chemical Engineering, RMIT University, Melbourne, Australia
nirdosha.gamage@rmit.edu.au, kevin.zhang@rmit.edu.au,
chunqing.li@rmit.edu.au, and sujeeva.setunge@rmit.edu.au

Abstract
Critical path method (CPM) is a deterministic technique that has been used extensively for managing projects because of its simplicity compared to other techniques. CPM analyses a project considering resources and time taken to complete each activity based on available data at the time of project planning. However, the predicted activity duration for a similar activity in a similar project will be different with the time, due to advancement in technologies, improved materials and various other factors. This paper reviews available methodologies used to predict activity duration and attempts to introduce probabilistic method to predict the project duration. The method uses simulation technique to generate probability distribution function based on contractor's previous performance on similar project. It enables project manager to detect level of uncertainty on the project duration and how extra resources could be launched with increased confidence to accelerate certain activity to complete the project on time.

Keywords
Critical path method, Probabilistic method, Project duration

1. Introduction

Critical path method (CPM) is an extension of bar chart and it has been used for project management since 1950s. The method offers project managers to estimate the project completion time effectively. CPM is a deterministic technique that analyses a project considering resources and time taken to complete each activity. The total project duration is obtained by adding durations of the longest sequence of tasks and by computing early start and early finish time of each activity, through a project network. Because of its simplicity, CPM is extensively used to calculate project duration. CPM should be customized to suit resources and goals of an individual organization and depending on individual project, considering calculated risks to reliably predict the duration (Stelth and Le Roy 2009). The uncertainty on project scheduling due to predictable risks is better addressed by considering the uncertainty related to the duration of the project activities (Pollack and Liberatore 2005; Barraza 2011). There are many disadvantages with the CPM. For example, CPM method is a deterministic method, which is not suitable for a dynamic construction environment in time forecasting. The project team's ability to reasonably predict the scope, schedule and cost of each project is far beyond control (Knoke and Garza, 2003). Too many interconnected activities could make the network diagram become complicated. Then, accurately identification of the critical chain may become difficult. When critical path and float changes, reallocation of workforce is difficult as several projects may run at the same time and one person may be working for more than one project or more than one activity in a project. Then, finding new people may be required. Or else, an individual who may have overloaded may find it difficult to finish all the tasks on time.

Activity duration is predicted using historical data that are available with a company and experience. Project managers use target estimated time and estimated committed time (planned) for each project based on duration estimates. The time difference between the planned and target duration is usually called project time allowance or contingency time of the project (Chapman and Ward 1997; Barraza 2011). Probabilistic approach can also be used instead of deterministic approach to estimate the project planned time or target. Duration variability is defined as random variables with specific probability distribution in the probabilistic approach whereas subjective criteria used by deterministic approach (Britney 1976, Barraza 2011). Barraza (2011) demonstrates a probabilistic approach to calculate project contingency time and its allocation at an activity level.

Program Evaluation and Review Technique (PERT) and Monte Carlo Simulation methods are probabilistic methods that have been used in many occasions to predict the project duration. PERT provides more optimistic project schedules because of the merger event bars (Lee and Arditi 2006). It considers only critical and non-critical paths and not-subcritical paths. Simulation techniques provides estimation of the probability that each activity become critical and is recommended for developing more realistic project plans (PMI 2004).

Project plans are developed by making number of assumptions about the technology or equipment used material and labour supply, key personal needed at reasonable costs (Kim et al. 2010). Even a well-developed project plan could also require changes during the delivery of the project due to various internal and external factors. That can affect the Critical Path (CP) logic during the process. These changes could be because of change in priorities, budget cuts, evolving regulations, potentially complex modifications, and change in the management of the organization during the duration of the project as well as unexpected weather conditions (Knoke and Garza 2003; Stelth and Le Roy 2009; Dawood 1997).

Earned value based forecasting method fundamentally measures, analyses and predicts schedule performance of a project in units of value (such as money, labour or percent completed). However, its scheduled performance index (SPI) or scheduled variance to forecast project duration can be fluctuates significantly (Vandevoorde and Vanhoucke 2006; Moselhi 2011). The method consider the ratio of the time (or cost) it took to complete all of the tasks up to a particular point in time of the project, compared to the time (or cost) which was estimated earlier (Alshibani and Moselhi 2012). If the actual time (or cost) is consistently greater than the planned numbers, it means that the method may consistently under-estimate the effort required. But it may revise the remaining estimates upward by a percentage that reflects actual experience to date.

In an effort to overcome limitations in the above method, probabilistic method is introduced for forecasting the duration. Also, how probabilistic method could be used to estimate total project duration more reliably is discussed.

2. Proposed Method to Predict Duration in the CPM

Project planned duration, (PPD) is the accumulation of predicted target activity duration (PAd) of each activity which can be expressed as: $PPD = \sum PAdi$. These durations can be calculated using either deterministic or probabilistic approach. In probabilistic scheduling, instead of using specific activity duration, duration variability is represented by random variables with specific probability distribution (Barraza 2011, Britney 1976). The reliability of the project been completed within the target time, is the difference between the Target duration and Actual duration, can be expressed as;

$$Re(Pp,t) = PPD - \sum AAdi = \sum PAdi - \sum AAdi$$

Where, $\sum PAdi$ is the predicted target duration of the project and $\sum AAdi$ is the actual duration of the project

Probability of project will complete within time, Pp(t), can be expressed as,

$$Pp(t) = P[Re(Pp,t) \geqslant 0]$$

Activity duration is widely influenced by management skills, productivity of the labour and plant, quality of material and weather. Based on previous experience of the company on how it handled similar projects in the past, future behaviour can be predicted. These historical data are analysed to find the most likely duration (mean) and the standard deviation (STD) or variance of that duration. These mean values as well as STD could follow different distribution functions. However, this paper makes simplified assumptions that activity duration follows a normal distribution to explain the method. Then, the Monte Carlo simulation (MCS) method is used to find the reliability of the predicted duration.

Once the target duration of each activity combined with maximum and minimum values are found, the Union Rule for Mutually Exclusive Events can be used to find the target project duration, minimum and maximum values. Also, the reliability of the project that will be completed within the target duration and maximum duration and its reliability can be computed too.

If all pairs of the events $A_1, A_2, \ldots, A_K$ are mutually exclusive,

Then the total project duration will be,
$$P(A_1 U\ A_2\ U\ldots, UA_K) = P(A_1) + P(A_2) + \ldots + P(A_K)$$

When any event intersected with any other event is the empty set, the union is the sum of the probabilities for each event.

3. Illustration of the method

3.1 Case1

A simple hypothetical project is used in this paper as in the Figure 2 to explain features of the proposed method. Figure 1 illustrates how each node is labeled in the diagram and what information is given. The critical path follows activities A1>A2>A4>A6 with 20 days duration. The path with A1>A3>A5>A6 takes 17days to complete that path.

Early Start		Early Finish
	Activity Name	Duration
	Standard Deviation	

Figure 1: description or label of a node

Then MCS was run to find likelihood of activity duration. It is assumed that activity duration follows normal distribution, for simplicity, to define the variability of it. Palisade Decision Tools software package (@Risks software) was used to undertake this analysis. The outputs of the developed distribution methods and forecasted duration are presented in Table 1 and illustrated in Figures 1 to 8 below.

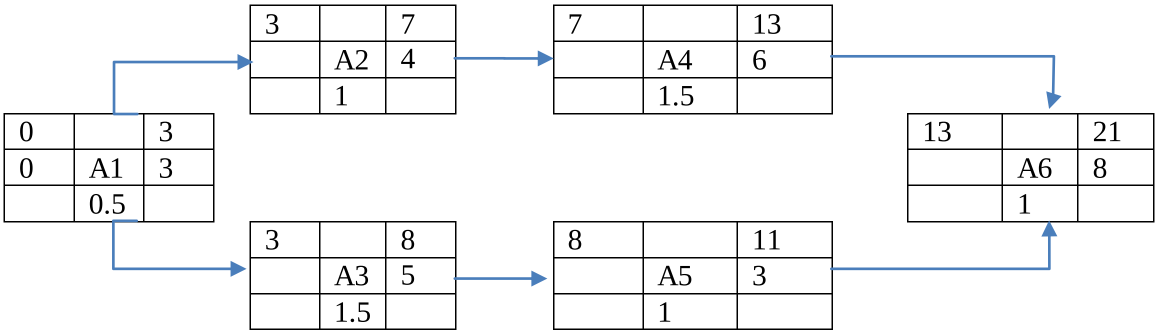

Figure 2: Network Diagram of the Hypothetical Project A

Table 1: Initial and predicted duration for case 1

Activity	Mean Duration	STD	Max Duration	Most likely (*95% confidence*) Maximum Duration (Simulated)
A1	3	0.5	3.5	3.82
A2	4	1	5	5.64
A3	5	1.5	6.5	7.46
A4	6	1.5	7.5	8.46
A5	3	1	4	4.64
A6	8	1	9	9.64
A1+A2+A4+A6	21			24.37

Figure 3 to 8 illustrates the probability density diagram and cumulative distribution of the duration of individual activity. Since it was assumed that the historical data on similar activities can fit into normal distribution, the probability of completing an activity within its target duration is 50%. However, maximum and minimum time required has changed as per their standard deviation. For example, the maximum duration for activity A1 (STD = 0.5) is 3.82 days with 95% confidence level and the average duration is 3 days. For the activity A5 (STD = 1), the maximum duration is 4.64 days with the same mean of 3 days. It demonstrates that, when STD of a duration is higher (higher risk of fluctuation), the predicted maximum duration becomes larger. Similarly, if the STD is shorter (low risk of fluctuation), the predicted duration is closer to the mean.

If with comparable conditions and with alike resources are launched, the maximum time required to complete the critical path (A1>A2>A4>A6) of the project is 24.37 days, with 95% confidence level (Figure 9). The maximum time that would take to complete the path A1>A3>A5>A6 would be 22.44 days (Figure 10). Therefore, the project manager can reliably take that the path A1>A2>A4>A6 as the critical path.

In real life constructions, delaying a project by a day would cost the company for its reputation, for extra salaries, plants and equipment as well as other resources that could be allocated for an another project or to mobilize a new project. If project manager needs to know, more confidently, how the project can be completed in 23 day and what extra resources may be required to achieve it, then he can plan to allocate extra resources as required. Also, if spending for this extra resource is economical or not to achieve the target can be evaluated to make the decision.

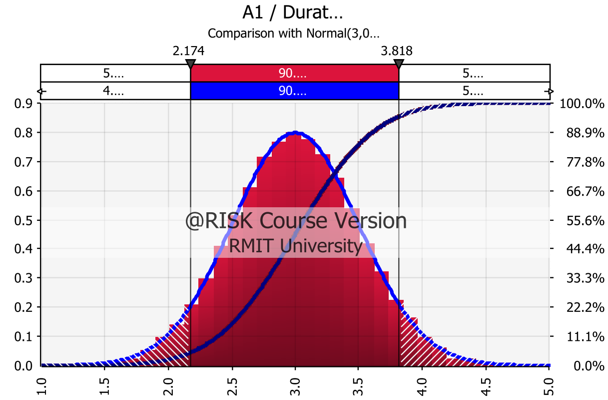

Figure 3: PDF for A1

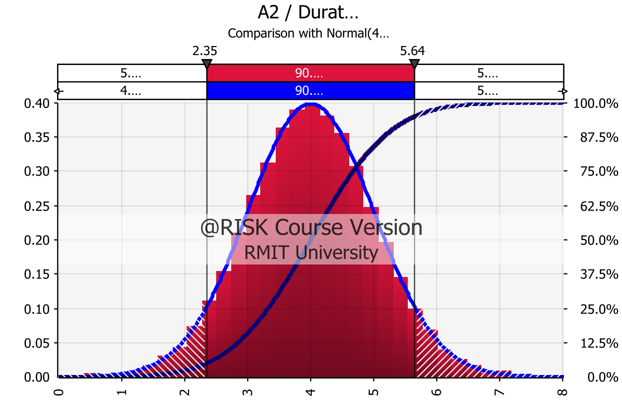

Figure 4: PDF for A2

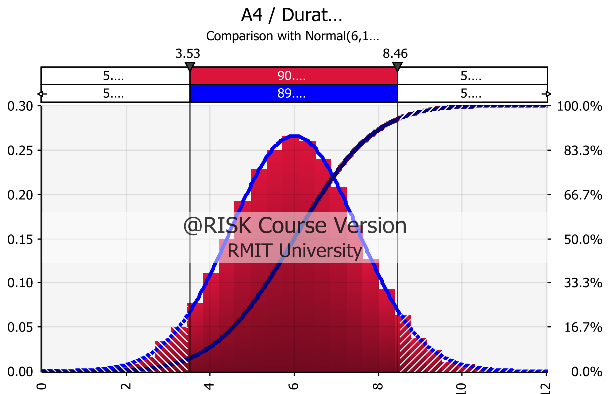

Figure 5: PDF for A4

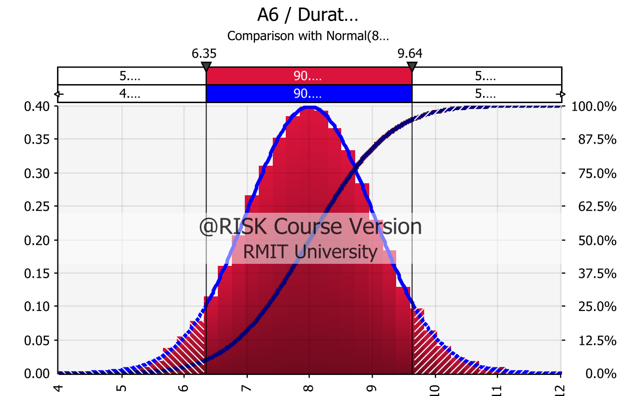

Figure 6: PDF for A6

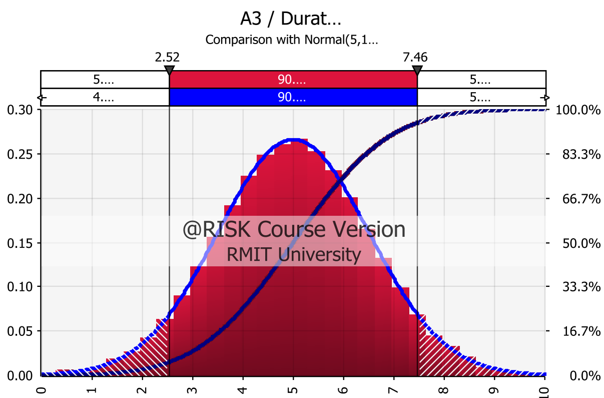

Figure 7: PDF for A3

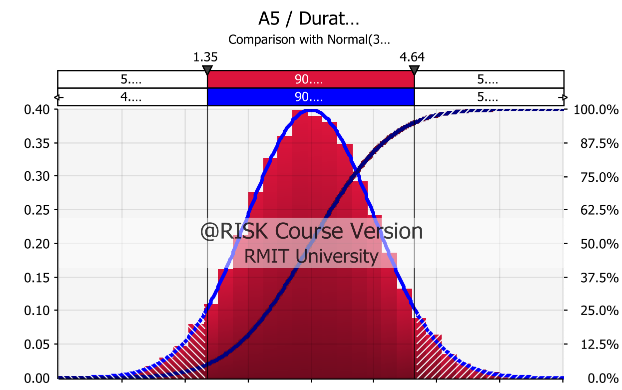

Figure 8: PDF for A5

The proposed method allows calculating the probability of achieving 23 days completion time and what extra action may require achieving it. Another simulation was done to calculate the probability of achieve target duration of 23 days. For this simulation, it was assumed that the duration of the activity A4, could be accelerated with minimum resource (Figure 11). It was found that the target project duration of 23 days can be achieved if A4 duration is 5 days, and with 91% confidence. Figure 11 illustrates that the distribution is normally distributed without any significant skew. If more resources can be allocated into the activity A4 in order accelerate the task, it can be forecasted with 91% confidence that the targeted 23 day duration can be achieved as long as other activities are completed within their target duration or prior to maximum predicted duration.

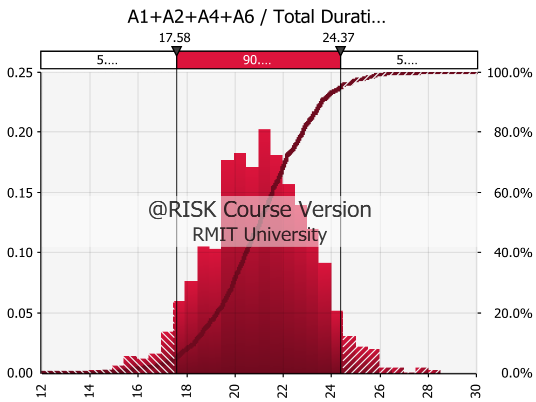

Figure 9: PDF for Critical Path

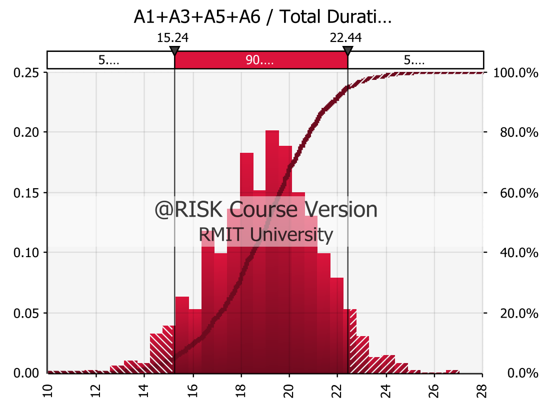

Figure 10: PDF for Alternative path

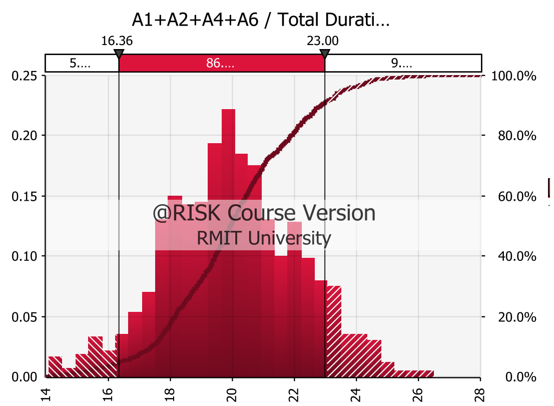

Figure 11: PDF for achieving target by accelerating A6

3.2 Case 2

Case 2 evaluates how a non-critical path could become a critical path of a project. The critical path of the Project B is B1>B2>B4>B6 with 34 days while the path B1>B3>B5>B6 would require 32 days to complete. Similar to Case 1, it was assumed that activity durations were found to be normally distributed (based or historical data). Figure 13 illustrates the PDF of the CP of the project (B1> B2>B4>B6) and Figure 14 shows the PDF of path B1>B3>B5>B6. The maximum time required to complete the critical path is 41 days with 95% confidence level. However, the probability of completing the alternative path within 41 days is 90%.

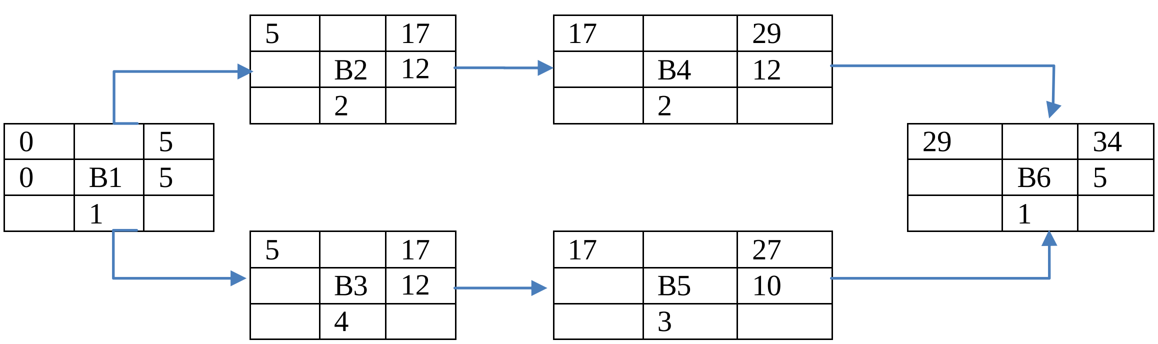

Figure 12: Network Diagram of the Hypothetical Project B

Therefore, there is a risk that the alternative path could become the critical path if activities in the path are not completed within their target time. That is because, the STD of activities B3 and B5 are greater compared to B2 and B4. However, the original deterministic critical path method doesn't have a capability to detect similar risk. Therefore, results that could be obtained from the proposed method are highly beneficial for project planning to reliably predict activity duration for CPM.

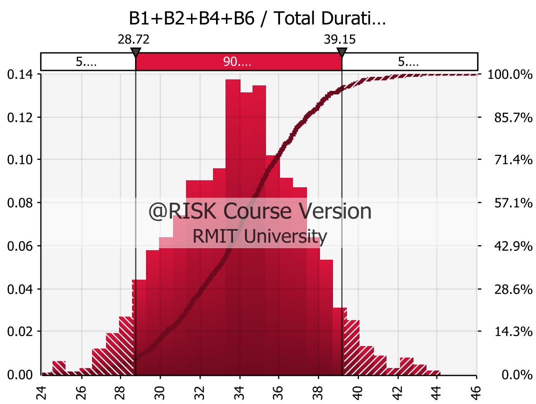

Figure 13: PDF for the original Critical Path

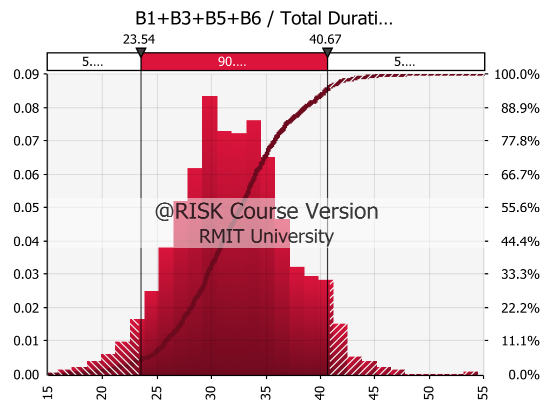

Figure 14: PDF for the alternative Path

In the construction industry the STD of an activity could change due to various factors such as weather conditions, material supply or equipment failure, unexpected groundwater conditions, defective design or design alterations, natural disasters, global economic fluctuation, changes in man power and the list could be continued. However, if the original historical data has realised and be aware of factors which have affected the STD, project manager can take proactive or precautionary measures to overcome those risk factors in order to complete the project on time.

5. Concluding Remarks

A reliability based method for evaluating activity duration of critical path method has been proposed and usability of it has been tested. It has been found that the variation (STD) of the target duration, which is calculated based of historical data, is the main factor that could contribute to the variation of target duration.

The method enhances contractors to predict activity durations, more reliably, at the project planning stage. However, the method is more suitable when historical data are available on similar projects or similar type of project activities to identify a suitable probability distribution, mean and STD. Also, the method is dependent on contractor's previous experience on similar projects, their resources as well as availability of material and plant hire etc. The duration could further alter considering predicted economic situations, financial cash flow or predicted weather or other risk factors. These factors could be considered when predicting STD, simulating STD separately as well as considering separate risk factors into the model improve reliability of predicted duration further.

6. References

Alshibani, A. and Moselhi, O. (2012). "Stochastic Method for Forecasting Project time and Cost", *ASCE: Construction Research Congress 2012*.

Barraza, G.A. (2011). “Probabilistic Estimation and Allocation of Project Time contingency”, *ASCE: Journal of Construction Engineering and Management*, 127, 259-265.

Britney R (1976) “Bayesian point estimation and PERT scheduling of stochastic activities”, *Manage.Sci.*, 22(9), 938-948.

Chapman, C.B. (1990). “A risk engineering approach to project risk management”, *International Journal of Project Management*, 8:(1).

Chapman, C., and Ward, S., (1997). “Project risk management: Processes, Techniques and Insights”, *John Wiley & Sons Ltd*., 408p

Dawood, N. (1997). “Estimating project and activity duration: a risk management approach using network analysis”, *Construction Management and Economics*, 16, 41-48.

Hillier, F.S. and Lieberman, G.J. (1986). Introduction to operations research, Project management with PERT/CPM: Simulation, 4th Ed. (cited in Lee D.E. 2005)

Lee, D.E, (2005). “Probability of Project Completion Using Stochastic Project Scheduling Simulation”, *ASCE: Journal of Construction Engineering and Management*, March 2005.

Lee, D.E., and Arditi, D., (2006). “Automated Statistical Anaylis in Stochastic Project Scheduling Simulation”, *ASCE: Journal of Construction Engineering and Management*, 132:268-277.

Kim, B.C. and Reinschmidt, K.F. (2010). “Probabilistic forecasting of Project Duration Using Kalman Filter and the Earned Value Method, *ASCE: Journal of Construction Engineering and Management*, August 2010.

Knoke, J.R. and Garza, J.d.I. (2003). Practical cost/scheduling for CIP management, *AACE International Transactions*.PM61.

Illumoka, A.A. (1987) A tolerance analysis approach to network scheduling for engineering project management, *International Journal of Production Research*, 25(4), 531-47.

Moselhi O., and Siao, X. (2011)., Trending and Forecasting in Construction Operations, *Proceedings of the 28th ISARC*, South Korea, 905-910

Pollack B. and Liberatore M (2005). Project Planning under uncertainty using scenario analysis, *Project Management Journal*, 36(1), 15-26.

Stelleth P. and Le Roy G (2009). Projects’ Analysis through CPM (Critical Path Method), *School of Doctoral Studies (European Union) Journal*, No.1

Vandevoorde S. and Vanhoucke M (2006). A Comparison of Different Project Duration Forecasting Methods using Earned Value Method, *International Journal of Project Management* 24(2006), 289-302

Zayed,2008Ahmad, I., and Azhar, S. (2005). “Implementing data warehousing in the construction industry: opportunities and challenges”. *Proceedings of the 3rd International Conference on Construction in the 21st Century (CITC-III)*, Athens, Greece, September 15-17, pp. 863-871.