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A FUZZY LOGIC APPROACH TO CRITICALITY IN SCHEDULING

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Abstract
The project duration compression is often a necessity that project managers have to deal with. The most common method is to compress the duration of tasks that are on the critical path, according to the Critical Path Method (CPM), starting from the one with the minimum compression cost per time. The characterization of a task as critical / non critical based on CPM lacks flexibility as it depends only on the amount of slack time of the specific task. In this way activities with non zero slack time but with characteristics that may be considered critical in the general sense of the term, as duration, cost and risks related to the task, are excluded of the compression process. The aim of the present paper is to generate a system based on multi-criteria analysis and fuzzy logic, that calculates the criticality degree of project activities and use this value as measurement for the choice of the most proper activity to be compressed.

Keywords: Fuzzy Logic, Criticality, Scheduling.

1. Introduction

The successful result of a project requires a complete coordination of all its functions/tasks. In practice, project managers give the maximum attention in cost and duration control, as these two parameters play the most important role in determining the development of the project. The need of changing the initial duration estimation of project tasks rises because of several problems that have as a result the delay of task completion and the lack of enough staff throughout the project, or because of the need of reducing the indirect costs. The problem of project crashing is usually confronted by compressing the duration of tasks that are critical according to the *Critical Path Method* (CPM), where critical tasks that compose the critical path have zero slack and thus no scheduling flexibility. A typical crashing procedure starts by compressing the duration of the critical task with the least compression cost per time. The characterization of a task as critical / non critical by defining only its slack time signifies a limitation in the perception of criticality as tasks that are not critical in the typical sense but which may be critical in a broader sense (due to a very small slack time or other characteristics that might influence the whole procedure of crashing), are excluded. These problems are tackled herein by generating a multi-criteria system, in which tasks are considered as critical based on their rating according to multiple criteria. The purpose of this approach is thus to generate such a system by setting a group of criticality criteria and using fuzzy logic in order to calculate the criticality degree of each task of the project, according to which decisions on the priority of task compression are taken.

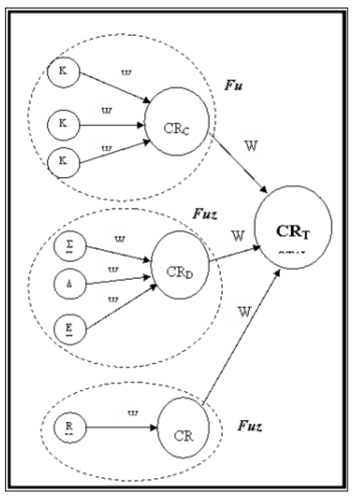
2. Methodology

There are three main categories of criteria that are chosen for defining the degree of criticality for a project task: cost, duration and risks related to the task. The steps to follow in order to calculate the criticality degree of a task in the present paper are: (i) Definition of the criticality criteria for each category; (ii) Calculation of the value for each criterion according to the project data; (iii) Use of these values as inputs to the calculation process of category criticality, through the three respective fuzzy systems that are generated for this purpose; and (iv) Calculation of the final criticality degree for each task through the function:

$$CR_{TOTAL} = \sum_i W_i CR_i$$

where CR_i is the criticality degree that comes as an output from the respective fuzzy system and W_i is a weight coefficient that depends on the type of the project (construction, IT services, etc.). For simplicity, in the present paper the coefficients are considered to represent Cost, Duration and Risk and are all considered equal to 0.33 (that is, $W_c=W_d=W_r=0.33$). The modeling of the entire system is depicted in Figure 1. The criticality criteria are further elaborated in the sequel.

Figure 1: Systemic description of the fuzzy system



(i) Cost

There are three criteria in this category which determine the criticality of a specific task: compression cost, overtime cost and equipment cost. The compression cost per time is the only criterion when project crashing according to CPM is applied. It is calculated by the function:

$$C_{Comp.} = \frac{C_c - C_n}{t_n - t_c},$$

where C_n is the task cost that refers to the initial duration t_n, and C_c is the task cost that refers to the compressed duration of the task t_c. In order to compare normalized values, the fraction

$$C_C = \frac{C_{comp.}}{Project\_Cost / Project\_Duration}$$

is used. The criticality degree of the task is increasing as the ratio C_C is decreasing.

The compression of an activity often demands the use of more resources or if there is no availability, the overtime use of the existing resources. The second case is more often and could lead to a very big increase of cost. The percentage of resource overtime usage and the cost of the specific resources may be a criticality factor. In order to define this degree we will use the ratio

$$C_{OVER} = \frac{C_{OVi}}{\sum C_{OV_i}}$$

where C_{OV_i} is the overtime cost of a task to the total overtime cost of the project and is calculated as

$$C_{OV_i} = \sum_i t_{OVER_i} \times C_{OVER_i}$$

where t_{OVER} is the overtime needed for a specific resource *i* and C_{OVER} is respective cost per hour. The sum of overtime costs of all the resources needed for the accomplishment of the activity is the overtime cost of the activity. The criticality degree of an activity is increasing as C_{OVER}. is decreasing.

A third factor that affects the cost criticality is the cost per use of the equipment and facilities until the accomplishment of an activity. This is typically an indirect cost which decreases as the duration of a certain task decreases. This factor is modeled by the ratio

$\frac{C_{MASH_i}}{\sum C_{MASH_i}}$, where C_{MASH_i} is the equipment cost for task *i*.

(ii) Duration

The criteria in this category that are used to determinate the criticality of a specific task are the total float (slack), the free float and the task duration. Float is a very valuable concept since it represents the scheduling flexibility or "maneuvering room" available to complete particular tasks. Activities on the critical path do not provide any flexibility for scheduling nor leeway in case of problems. For activities that have some float, the actual starting time might de chosen to balance work loads over time, to correspond with material deliveries, or to improve the project's cash flow. The total float is the maximum amount of delay which can be assigned to any activity without delaying the entire project. According to CPM, total float is the only criterion for characterizing a task critical / non critical. In the present paper we use as a criticality factor the ratio TF_i/PD, where PD is the project duration. The lower this ratio, the higher the criticality degree of a task gets. Free float is the amount of delay which can be assigned to any activity without delaying subsequent activities. When there are no subsequent tasks free float is equal to total float. In the present paper we will use as a criticality factor the ratio FF_i/PD. The lower this ratio, the higher the criticality degree of a task gets. Long duration tasks tend to affect the project more than short duration tasks. If, for example, we have three tasks of 120 days, 10 days and 5 days respectively, it is understood that the compression of the 10d and 5d duration tasks will not affect the project time as the compression of the 120d duration task would have. We will use the ratio D_i/PD as a criticality factor; the higher the ratio, the higher the criticality degree of a task.

(iii) Risk

A risk is any factor that may potentially interfere with successful completion of the project. A risk is not a problem; a risk is the recognition that a problem might occur. By recognizing potential problems, the project manager may succeed in avoiding a problem through proper actions. The procedure that a risk management team uses to manage risks is defined in the planning stage, documented in the project plan, and then executed throughout the life of the project. Risk management deals with the following risk phases: (i) Risk identification; (ii) Risk analysis and quantification; (iii) Risk mitigation planning; and (iv) Risk response.

In order to define a risk measure that might be used as criticality criterion, we first identify risks related to a project and then develop lists that allow managers to assign a score to the probability of occurrence and the degree of impact that these risks have on the project goals. The ratio of the score a task gets to the maximum score it could get can be a criticality criterion; the lower this ratio, the higher the criticality degree of the task, since its compression will not probably cause problems to the goals of the project. This method, although simple, has the disadvantage of needing great experience by those who assign the impact and probability grades. In the present paper we follow a different path for defining the risk criticality of an activity. We try to identify the characteristics of an activity that can be risk factors, so whenever a risk occurs during the specific task, the impact on the goals of the project will be greater than on any other task that does not have these characteristics. Task cost, duration, use of innovative technology and other characteristics can be defined as risk factors that determine the risk level of an activity. The ratio of the score a task will get to the maximum score will be the criticality criterion that refers to risk. Such a scoring list is presented in Table 1.

Risk factors	**high**	**If yes score . 5**	**medium**	**If yes score . 3**	**low**	**If yes score. 1**	**none**	**If yes score. 0**
Estimated task cost	>20% project cost		10-20% project cost		5-10% project cost		<5% project cost	
Precision of cost estimation	Estimation precision <50%		Estimation precision 50-85%		Estimation precision 85-95%		Estimation precision >95%	
Use of new technology	Technology in development		Very new technology		Technology available for several years		Technology proven and available for many years	
Demand of highly trained staff	demand >20%M.O trained staff		demand 10-20% M.O trained staff		demand 5-10%M.O trained staff		demand <5%M.O trained staff	
Influence on subsequent tasks	>4 subsequent tasks		2-4 subsequent tasks		1-2 subsequent tasks		No subsequent tasks	
Influence by political, economical, judicial changes	Increase >20% estimated cost		increase 10-20% estimated cost		Increase <10% estimated cost		Insignificant increase estimated cost	
Percentage of rare resources use	Is >30% task resources		Is 15-30% task resources		Is 5-15% task resources		Is <5% task resources	
Demanded quality level	Very high		Medium		Low		Very low	
Task estimated duration	>20% project duration		10-20% project duration		5-10% project duration		<5% project duration	
Precision of duration estimation	Estimation precision <50%		Estimation precision 50-85%		Estimation precision 85-95%		Estimation precision >95%	

Table 1: **Risk Analysis Table.**

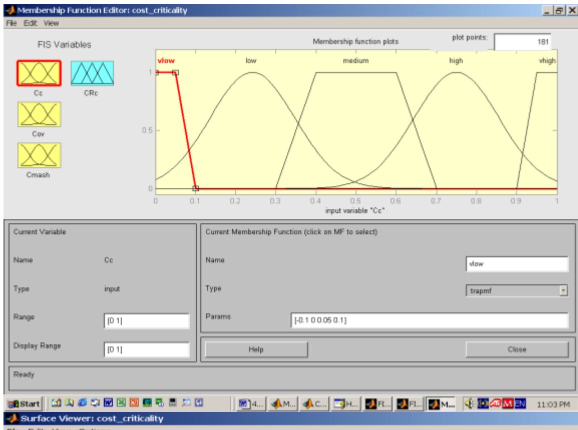
3. Fuzzy systems

Over the last thirty years there has been an effort to develop new non-conventional control techniques, which are based on the understanding and reproduction of human intelligence. Intelligent Control Systems, as they are called, merge ideas and techniques from several sciences such as business management, psychology, information science, communications and conventional control theory in order to develop methods for better management of fuzziness, inductive reasoning and connectionism or parallel distributed processing. They are based on the knowledge and experience of the man-controller and do not demand the in depth knowledge of the procedure under control, as happens in conventional control systems. Fuzzy logic is one of the intelligent control techniques. Some of the basic terms of fuzzy logic are: (i) *Fuzzy variable*: name of a fuzzy set; (ii) *Membership function*: functions that define the fuzzy set's shape; (iii) *Fuzzy operators*: represent the union, intersection and complement of two fuzzy sets; (iv) *Hedges:* play the same role as adverbs and adjectives in English. They transform the shape of a fuzzy set; and (v) *Implication*: method of functional tie between the degree of truth in related fuzzy regions.

In the present paper we develop three systems for calculating the criticality degree of a task, which are based on fuzzy logic. In order to do so, we use Fuzzy Toolbox of Matlab7. The basic steps of a fuzzy algorithm are: (i) The fuzzification of real-time procedure data; (ii) The application of fuzzy operators and determination of implication method; (iii) The aggregation of outputs; and (iv) The de-fuzzification of fuzzy data. In order to build the fuzzy systems, we define the linguistic variables (in this case the variables are represented by the criticality criteria), determine the membership functions for each variable, write the rules of the system and determine the methods of implication, aggregation and de-fuzzification. These inputs and outputs are:

	INPUTS	**OUTPUTS**
FUZZY 1 - Cost	$C_C = \frac{C_{comp.}}{Project\_Cost / Project\_Duration}$	CR_C
	$C_{OVER} = \frac{C_{OVi}}{\sum C_{OV_i}}$	
	$C_{MASH} = \frac{C_{MASH_i}}{\sum C_{MASH_i}}$	
FUZZY 2 - Duration	St = TF_i/PD	CRd
	Sf = FF_i/PD	
	DU = D_i/PD	
FUZZY 3 - Risk	RR= $\frac{totalscore}{\max .score}$	CRr

The membership functions used in these systems are trapezoidal, triangular and gaussian functions, and each variable is represented by five functions (very low-low-medium-high-very high). The membership function for C_C is shown in Figure 2.



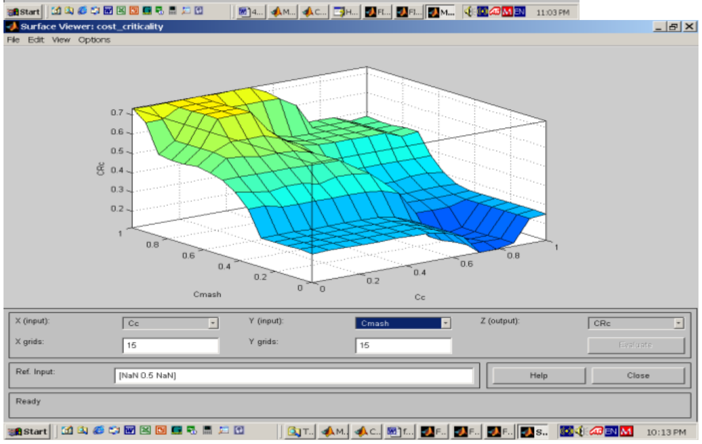


Figure 2: Membership Function **Figure 3: Decision Surface**

All systems use the Mandani fuzzifier, the de-fuzzification method is the centroid method, the operators AND and OR are represented by the min and max rule respectively. There are 125 control rules on the knowledge base of fuzzy 1, 95 rules on the knowledge base of fuzzy 2 and 5 rules on the knowledge base of fuzzy 3. A snapshot of these rules follows:

IF C_C= very low and C_{OVER}= very low and C_{MASH}= very high THEN CR_C= very high
IF C_C= very low and C_{OVER}= low and C_{MASH}= low THEN CR_C= medium
IF St= very low and Sf= very low and DU= very high THEN CR_C= very high
IF St= very low and Sf= medium and DU= very high THEN CR_C= high
IF RR= very low THEN CR_R= very high

The surface viewer of Fuzzy Toolbox allows the visualization of how the output of each system is influenced by any combination of inputs. The surface of CR_C influenced by C_C, C_{MASH} is shown in Figure 3.

4. Conclusions

We generated a multi-criteria system based on fuzzy logic in order to calculate the criticality degree of project's activities. As one can see that there may exist other factors that influence the sense of criticality in a project and must be taken under consideration when a decision must be made about the project compression. This specific system can be enriched with more criticality factors, a better determination of W_i coefficients, based on historical data or other options for the membership functions. The system is modular and flexible. Preliminary results demonstrate the feasibility of the approach and its superiority over traditional approaches.

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